

## Refined Scientific Requirements for the PTCS

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### Abstract

Multi-stage scientific requirements for the PTCS were developed prior to the Conceptual Design Review in 2003 April: The GBT should deliver astronomically useful performance (1) up to the atmospheric cutoff at 52 GHz by 2003 October, (2) up to 86 GHz by early 2005, and (3) up to 115 GHz by 2005 October. The most fundamental performance requirements needed to reach 52 GHz are pointing errors  $\sigma_2 = 2''.8$  and effective surface errors  $\epsilon = 0.46$  mm. The corresponding 86 GHz requirements are  $\sigma_2 = 1''.7$  and  $\epsilon = 0.28$  mm, and the 115 GHz requirements are  $\sigma_2 = 1''.3$  and  $\epsilon = 0.21$  mm. These requirements have been refined and expanded on the basis of new instrumentation, observations, and analysis by the PTCS team through 2003 November. Separate limits for the larger error components have been specified. New requirements for operational efficiency have been used to determine tracking timescales and performance under unfavorable environmental conditions.

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## History

**27.1** 2003 Nov 22. Original version (J. J. Condon).

**27.2** 2003 Nov 27. Revised version (J. J. Condon).

### 1. GBT Science Summary

The GBT is a general-purpose telescope intended to cover the entire frequency range accessible through the atmosphere at Green Bank. The universal scientific requirements for high aperture efficiency, low sidelobes, and small pointing errors all imply GBT dimensional tolerances which are proportional to the observing wavelength  $\lambda$ . Thus the PTCS is driven by the goal of making astronomically useful observations at the shortest wavelengths accessible through the atmosphere at Green Bank.

The major scientific programs envisioned for the GBT and their frequency ranges are illustrated in Figure 1 (NRAO 2000) along with the “stop bands”  $52 \lesssim \nu \lesssim 68$  GHz and  $\nu \gtrsim 117$  GHz where the atmosphere is nearly opaque. The GBT already works quite well at the H<sub>2</sub>O  $\nu \approx 22$  GHz line frequency. The PTCS goal for the 2003/4 winter observing season is to make the GBT an astronomically useful telescope up to the 52 GHz ( $\lambda \approx 6$  mm) atmospheric cutoff. In the 68 to 117 GHz window, our ultimate goals are good performance at 86 GHz ( $\lambda \approx 3.5$  mm) and usable performance up to the 115 GHz ( $\lambda \approx 2.6$  mm) CO  $J = 1 - 0$  line.

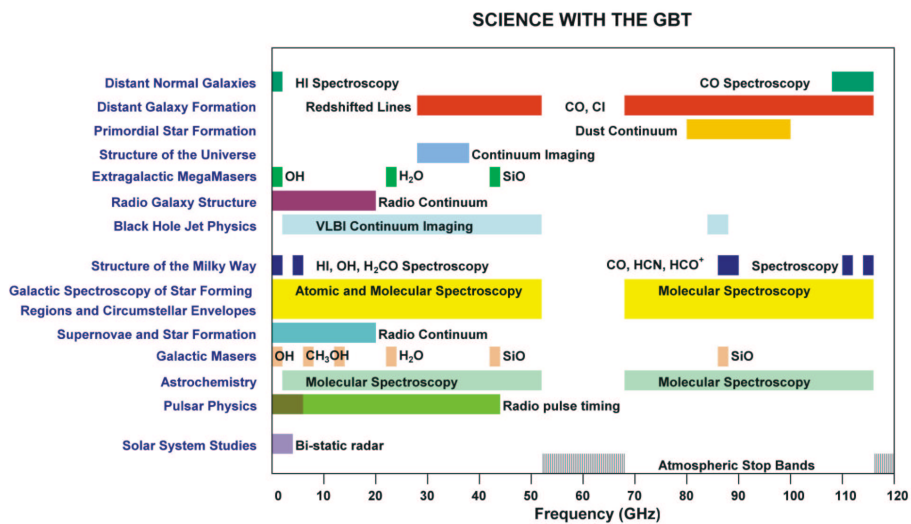


Fig. 1.— Frequency coverage of GBT science (NRAO 2000). Abscissa: Frequency (GHz).

Several different observing scenarios (e.g., source tracking, mapping, VLBI) were considered in the original analysis of scientific requirements (Condon 2003a). Source tracking appears to be the most demanding, so we have concentrated on meeting the performance requirements for an astronomer who wants to track a weak point source whose position is known in advance. To make efficient use of the GBT when the atmospheric transparency is good, we must minimize the calibration overhead from offset pointing as well as high-frequency observing time lost to high winds and thermal stresses.

## 2. Source Tracking, Sensitivity, and Flux Errors

Errors in tracking a point source cause both a loss of mean sensitivity and fluctuations in measured flux densities. The GBT beam is a circular Gaussian of diameter

$$\left(\frac{\theta}{740 \text{ arcsec}}\right) \approx \left(\frac{\text{GHz}}{\nu}\right) \quad (1)$$

between half-power points. The normalized power gain  $g$  can be written as

$$g(\rho) = \exp\left[-4 \ln(2) \left(\frac{\rho}{\theta}\right)^2\right], \quad (2)$$

where  $\rho$  is the angular displacement from the beam center. The probability distribution of  $g$  on the position of a point source depends on the probability distribution of the tracking error  $\rho$ . Numerous observations of strong calibration sources at 9 GHz during the fall of 2003 (Condon 2003b; Balser et al. 2003) confirm that the GBT pointing errors have roughly Gaussian distributions in the natural telescope coordinates, azimuth  $A$  and elevation  $E$ . If the errors  $\Delta A$  and  $\Delta E$  are uncorrelated, the tracking error

$$\rho = [(\Delta A)^2 + (\Delta E)^2]^{1/2} \quad (3)$$

has a Rayleigh distribution with mean squared error

$$\langle \rho^2 \rangle \equiv \sigma_2^2 = \int_0^\infty \rho^2 p_\rho(\rho) d\rho = \sigma_A^2 + \sigma_E^2. \quad (4)$$

We define ‘‘pointing error’’ or ‘‘tracking error’’ to be the rms offset in two dimensions,  $\sigma_2$ .

The differential probability distribution of on-source gain  $g$  was calculated by Condon (2003a) in terms of the pointing error expressed as a fraction  $f$  of the beamwidth

$$f \equiv \left(\frac{\sigma_2}{\theta}\right). \quad (5)$$

In most cases the sensitivity lost through the reduction of mean gain  $\langle g \rangle$  can be recovered by additional integration. The fractional increase  $\Delta\tau$  in integration time needed to compensate for tracking errors is plotted as a function of  $f$  in Figure 2. The rms fractional uncertainty  $\sigma_s$  in estimates of source flux density  $S$  caused by pointing errors is also shown in Figure 2. The corresponding distribution  $P_s$  of the unbiased flux-density estimator  $s = g/\langle g \rangle$  caused by pointing errors is very skewed (Figure 3). The long negative tails of  $P_s$  particularly hamper detection experiments by greatly increasing the integration times required to set reliable upper limits. The bottom line is that *fluctuations* in gain have much worse consequences than the *mean* gain loss, in terms of both data quality and operational efficiency.

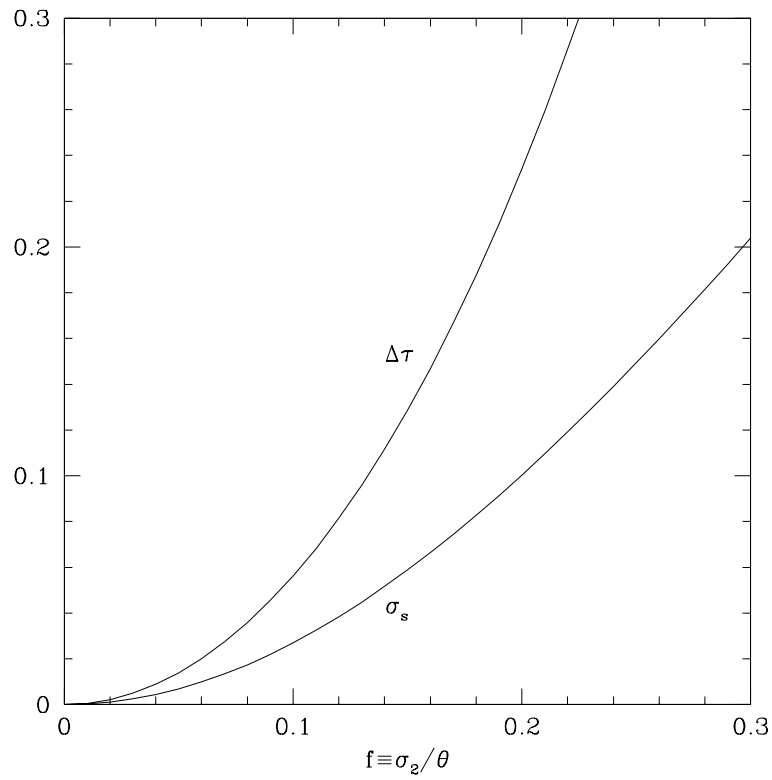


Fig. 2.— Tracking errors cause a mean sensitivity loss which may be recovered via increased integration. They also cause uncorrectable flux-density calibration uncertainties. Abscissa: RMS tracking error in beamwidths (dimensionless). Ordinate: Fractional increase in integration time  $\Delta\tau$  needed to compensate for sensitivity loss (dimensionless) or fractional flux error  $\sigma_s$  (dimensionless).

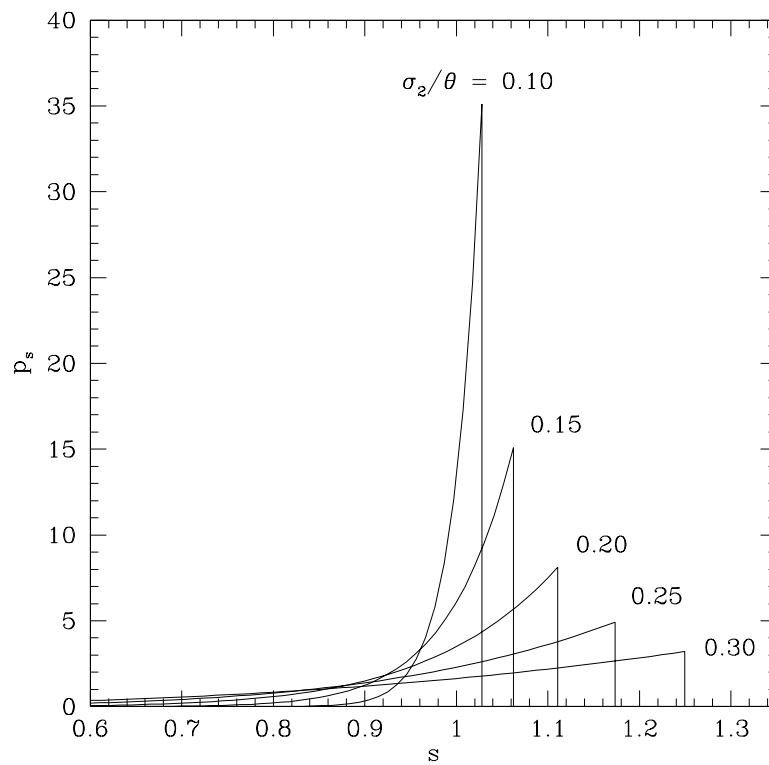


Fig. 3.— Distributions of measured flux densities resulting from pointing errors. The long negative-going tails adversely affect the reliability of detection experiments, significantly increasing the integration time needed to reach a given limit. Abscissa: Ratio  $s$  of observed to true flux density (dimensionless). Ordinate: Probability  $P_s$  (dimensionless). Parameter: RMS tracking error in beamwidths  $f = \sigma_2/\theta$  (dimensionless).

## 2.1. Tracking Accuracy Requirements

If we require fluctuations  $\sigma_s \lesssim 0.1$  (10% rms flux errors) for astronomically “usable” performance, then the largest acceptable tracking error is

$$f \equiv \frac{\sigma_2}{\theta} \approx 0.20. \quad (6)$$

In words, the rms offset between the commanded and actual telescope position should not exceed 0.20 beamwidths. For “good” performance (5% rms flux errors), the normalized pointing error should not be more than

$$f \approx 0.14. \quad (7)$$

The rms errors corresponding to usable and good pointing at 52, 86, and 115 GHz are listed in Table 1. We met the PTCS requirement for usable pointing at 52 GHz by 2003 December. By early 2005, we should have usable pointing at 86 GHz. By October 2005, the goals are good pointing at 86 GHz and usable pointing at 115 GHz.

We must use offset tracking to satisfy the pointing requirements listed in Table 1. Fortunately, the GBT is so sensitive that thousands of strong sources having accurate ( $\sigma_2 \approx 0''.7$ ) absolute positions can be used as offset pointing and focusing calibrators (Condon & Yin 2001). A suitable calibrator can be found within  $3^\circ \approx 0.05$  radians of most celestial positions north of declination  $\delta = -40^\circ$ . If the measured azimuth and elevation offsets of the calibrator can be applied in real time, the GBT needs only have good differential pointing accuracy over angular shifts of several degrees and maintain accurate tracking over the time interval between successive observations of the calibrator. A practical PTCS requirement for 2003 October was developing new observing software so that pointing/focusing calibration scans can be made quickly and easily, analyzed automatically in real time, and applied automatically to update the GBT azimuth and elevation offsets.

Our all-sky pointing runs indicate that pointing errors increase at low elevations  $E \lesssim 15^\circ$ . It is unlikely that good 3 mm observations can be made at lower elevations from Green Bank because the atmospheric zenith opacity in good conditions is  $\tau_z \approx 0.1$ . Scaling  $\tau(E) = \tau_z \operatorname{cosec}(E)$  implies  $\tau \gtrsim 0.4$  at  $E \lesssim 15^\circ$ . Also, the  $\delta = -40^\circ$  limit of our calibration source list means that calibrators cannot be guaranteed below  $E \approx 13^\circ$ . Thus the PTCS pointing requirements need not apply below  $E = 15^\circ$ .

## 2.2. Pointing Calibrators

The GBT pointing calibrators currently have rms errors  $\sigma_2(\text{cal}) \approx 0''.7$  in their absolute positions (Condon & Yin 2001). Their positions were measured from the NVSS (Condon et al. 1998) images having  $\theta = 45''$  resolution because the sensitivity of the NVSS to extended structure and confusing sources is essential for calibration at lower frequencies where the single-dish beamwidth is large. For the GBT,  $\theta = 45''$  at 16 GHz. The current calibrator position errors are insignificant at 52 GHz, but they contribute up to 1/3 of the acceptable rss (root sum square) pointing errors  $\sigma_2^2$  at 86 and 115 GHz (Table 1). At higher frequencies the GBT beam is small enough ( $\theta \sim 7''$ ) that far more accurate positions [ $\sigma_2(\text{cal}) < 0''.2$ ] measured by interferometers with smaller synthesized beams can and should be substituted. A conservative PTCS requirement is that calibrator position errors contribute no more than 1/4

of the total tracking error, or  $(1/4)^2$  of the tracking variance:

$$\sigma_2^2(\text{cal}) = \left(\frac{\sigma_2}{4}\right)^2. \quad (8)$$

Most of the necessary data already exist in the VLA archives or the FIRST survey catalog (Becker et al. 1995). The PTCS group should extract and analyze the archive data before the first 3 mm observations are made in early 2005, and new X-band snapshots with the VLA D-configuration may be needed to fill in the gaps.

### 2.3. Tracking Time

Very few astronomical observations actually require continuous tracking of a source (bistatic planetary radar is one example), so the principal disadvantage of offset pointing is lowered operational efficiency. To minimize the calibration overhead, we have (somewhat arbitrarily) set the maximum acceptable fraction  $\beta$  of time spent calibrating to be

$$\beta = 0.2. \quad (9)$$

This scientific requirement has consequences for the minimum acceptable time interval  $\Delta t$  between successive calibrator scans in units of the duration  $t_{\text{cal}}$  of a calibration observation:

$$\Delta t = \left(\frac{t_{\text{cal}}}{\beta}\right). \quad (10)$$

We currently calibrate using “jack” scans consisting of two elevation scans (one forward and one back), two azimuth scans, and a single axial focusing scan. The time required to slew from a program source to its nearby calibrator, make a jack scan, and return to observing is  $t_{\text{cal}} \approx 5$  min. The minimum time over which the GBT must be able to maintain accurate offset tracking is thus

$$\Delta t = \left(\frac{5 \text{ min}}{0.2}\right) \approx 0.5 \text{ hr}. \quad (11)$$

We have provisionally adopted  $\Delta t = 0.5$  hr as the PTCS requirement for tracking time. Since higher operational efficiency is always nice, we have also set  $\Delta t = 1$  hr as a PTCS “goal” for tracking time; it implies an overhead of only  $\beta \approx 0.08$ . To the extent that we can shorten the calibration time  $t_{\text{cal}}$ , we may be able to shorten these tracking times in the future.

Astronomical sources move across the sky at rates up to  $15^\circ \text{ hr}^{-1}$ , so our minimum  $\Delta t$  implies that the GBT must have good offset pointing accuracy and small blind pointing gradients over angular scales up to  $[15^\circ/\Delta t(\text{hr})]$  as well as good blind pointing stability over time  $\Delta t$ . These blind pointing requirements are specified in Section 3.

### 2.4. Wind

High winds will cause unacceptable pointing errors with the exposed GBT. The relevant scientific requirement is to minimize the fraction of high-frequency observing time lost to wind.

Wind produces pointing variations (Constantikes 2003a) that are well sampled over the 40 s time differences between forward and backward scans in azimuth and elevation made during “jack” scans of calibration sources. The rms ( $F - B$ ) azimuth and elevation differences increase with wind speed  $|v|$ , and the observed wind contribution is about (Condon 2003b)

$$\sigma_1(F - B) \approx 1'' \left( \frac{|v|}{2.5 \text{ m s}^{-1}} \right)^2 \approx \sigma_2(\text{wind}). \quad (12)$$

We have made the approximation that the rms difference between two one-dimensional (azimuth or elevation) positions is the same as the rms two-dimensional pointing error  $\sigma_2(\text{wind})$  contributed by wind. Equation 12 is consistent with the RSI calculation (RSI 2000) for  $s = 6 \text{ m s}^{-1}$  winds gusting to  $7 \text{ m s}^{-1}$ . RSI estimated two-dimensional nonrepeatable pointing errors  $\sigma_2(\text{wind})$  between  $4''$  and  $12''$  depending on GBT pose and wind direction. Equation 12 indicates  $\sigma_2(\text{wind}) \approx \sqrt{2}\sigma_1(\text{wind}) \approx 8''$  at  $s = 6 \text{ m s}^{-1}$  when averaged over a wide range of poses.

To estimate the maximum acceptable wind speeds for high-frequency observing, we constrain the pointing variance contributed by wind to be 1/4 of total pointing variance  $\sigma_2^2$ , or

$$\sigma_2^2(\text{wind}) = \left( \frac{\sigma_2}{2} \right)^2. \quad (13)$$

These “current wind” speeds listed in Table 1 are the highest wind speeds satisfying both Equation 12 and Equation 13; that is, the highest wind speeds for acceptable pointing when no attempt has been made to correct the pointing for wind-induced errors.

Since wind forces are proportional to the square of wind speed, we cannot hope to achieve our pointing goals in extremely windy conditions. Rather, we should try to minimize potential high-frequency observing time lost to wind. The cumulative distribution of wind speeds measured by the the “Weather 2” station near the GBT is shown in Figure 4 (Constantikes, private communication). This distribution is somewhat pessimistic because the wind speeds are generally lower during the relevant periods of good atmospheric transparency. The median wind speed is  $\approx 4.5 \text{ mph} \approx 2.0 \text{ m s}^{-1}$ . The wind speed is less than  $5.6 \text{ mph} \approx 2.5 \text{ m s}^{-1}$  for 60% of the time, less than  $7.8 \text{ mph} \approx 3.5 \text{ m s}^{-1}$  for 75% of the time, and less than  $11.2 \text{ mph} \approx 5.0 \text{ m s}^{-1}$  for 90% of the time. We use these three speeds to define “benign,” “normal,” and “extreme” wind conditions, respectively.

Table 1 indicates that the GBT is already usable at 52 GHz under “benign” conditions, the PTCS requirement for 2003 October. For early 2005 the PTCS requirements will include (1) usable pointing at 52 GHz under “normal” wind conditions (75% of the time) and (2) good pointing at 52 and usable pointing at 86 GHz under “benign” wind conditions (60% of the time). By October 2005 we should have good pointing at 86 GHz and usable pointing at 115 GHz under “benign” conditions (60% of the time).

## 2.5. Temperature

Temperature gradients across the exposed GBT cause thermal deformations which account for at least half of the nonrepeatable pointing errors during benign wind conditions (Constantikes 2003b). Fluctuating thermal gradients may appear during sunrise, sunset, or throughout a sunny day if the GBT is

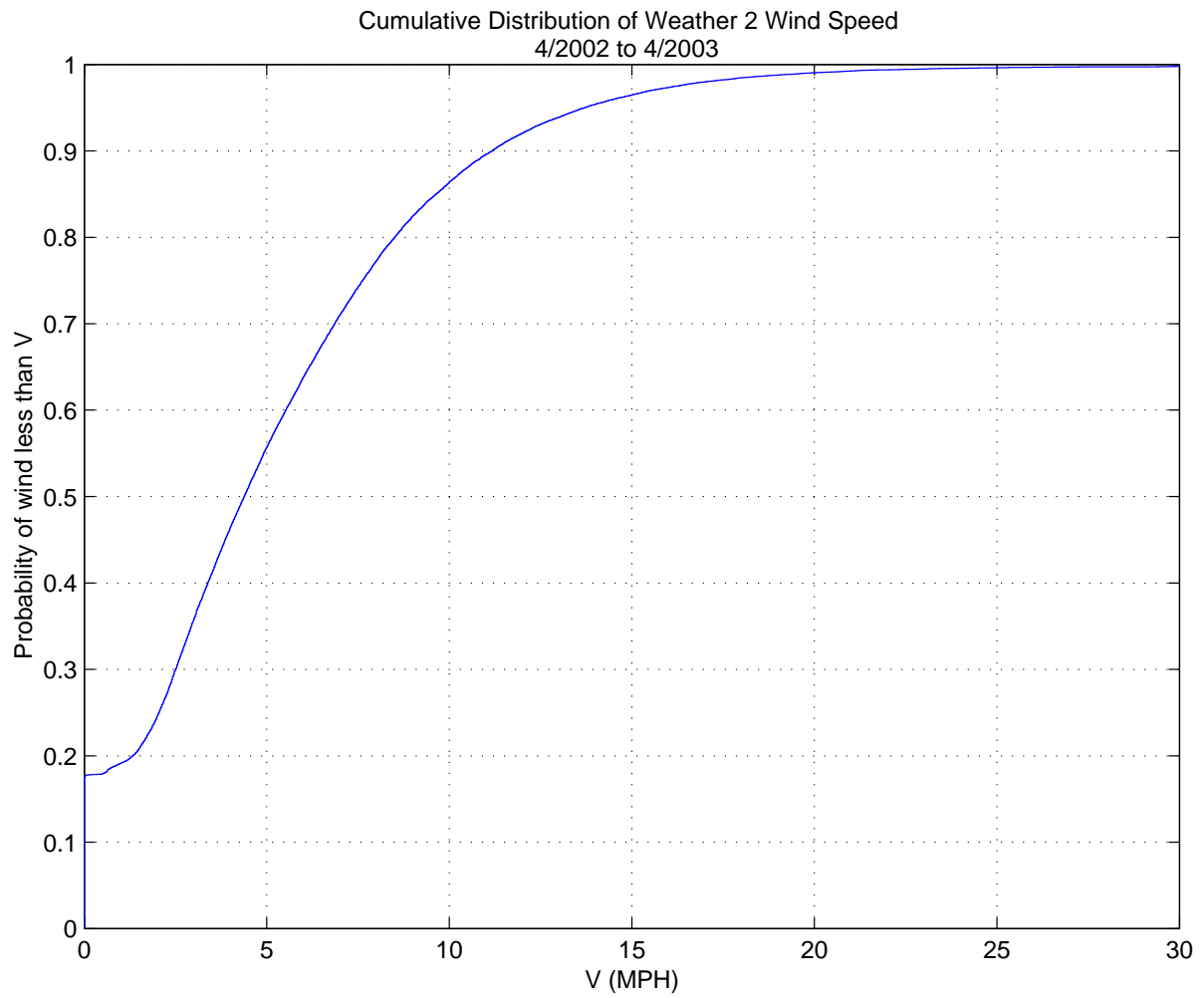


Fig. 4.— The cumulative distribution of wind speeds measured near the GBT.  $1 \text{ m s}^{-1} \approx 2.24 \text{ mph}$ .

slewed rapidly over the whole sky. Again, it makes sense define the scientific requirement in terms of operational efficiency: we want to minimize the loss of otherwise acceptable high-frequency observing time. Assigning up to 1/4 of the total pointing variance to thermal effects sets the maximum  $\sigma_2(\text{temp})$ :

$$\sigma_2^2(\text{temp}) = \left(\frac{\sigma_2}{2}\right)^2. \quad (14)$$

We do not yet have a simple model relating GBT pointing to temperatures measured on the structure, so we are not prepared to define “benign”, “normal”, and “extreme” thermal conditions as we have done for wind (Sec. 2.4). Our recent pointing results (Constantikes 2003b) show that we can satisfy Equation 14 at night at 52 GHz, and probably during the day for offset-tracking scans of a single source. The ultimate PTCS requirement should be to satisfy Equation 14 at 115 GHz during the night, and our goal would be daytime operation to at least 86 GHz.

Pointing errors can be reduced significantly with the use of temperature data from many points on the GBT (Constantikes 2003b). An immediate goal of the PTCS is to calculate these corrections and correct GBT pointing in real time.

### 3. Blind Pointing

The tracking accuracy requirements (Eqs. 6 and 7) do not imply such strict requirements for the “blind” or absolute pointing accuracy. Even so, the blind pointing must be good enough so that (1) the nearest pointing calibration source can be located easily and (2) gradients in the blind pointing errors do not degrade offset tracking in less than the tracking time  $\Delta t = 0.5$  hr.

#### 3.1. Calibrator Acquisition

We require that the calibrator pass through the beam half-power circle on the first arm of a jack scan at least 90% of the time, so that it is rarely necessary to hunt for a calibration source the first time it is used. This requires accurate blind pointing in *either* azimuth *or* elevation but not both. Currently the GBT blind pointing is significantly better in  $A$  than in  $E$ , so the first scan should be in elevation at fixed azimuth. The azimuth error distribution is Gaussian so our requirement implies

$$\int_{-\theta/2}^{+\theta/2} \frac{1}{(2\pi)^{1/2}\sigma_A} \exp\left(-\frac{A^2}{2\sigma_A^2}\right) dA = 0.9. \quad (15)$$

Numerically,

$$\left(\frac{\sigma_A}{\theta}\right) \approx 0.30. \quad (16)$$

If necessary, this specification might be relaxed by the use of fancier search methods, such as “nautilus” spiral scans, to new pointing calibrators.

### 3.2. Blind Pointing Gradients and Tracking Stability

We require that position and time gradients in the blind pointing errors be small enough for offset pointing using nearby calibrators and offset tracking over periods of time  $\Delta t \approx 0.5$  hr (Eq. 11). At the sidereal rate, the time interval  $\Delta t$  equivalent to the typical distance to the nearest calibrator  $\Delta\phi \approx 3^\circ$  (Condon & Yin 2001) is only 12 minutes. Cycling between source and calibrator more frequently than once every  $\Delta t \approx 12$  min is probably not worth the overhead in lost observing time. The angle  $\Delta\phi$  on the sky over which changes in the blind pointing error should be much smaller than the acceptable tracking error  $\sigma_2$  is

$$\left(\frac{\Delta\phi}{\Delta t}\right) \approx \left(\frac{15^\circ}{\text{hour}}\right), \quad (17)$$

where  $\Delta t$  is the time between calibration scans. We make the approximation that the tracking error increases linearly with the angular separation  $\Delta\phi$  between the  $(A, E)$  coordinates of the calibration scans and the track. Thus, at the end of a  $\Delta t = 0.5$  hour track,  $\Delta\phi \lesssim 7.5^\circ$ . Offset pointing does not remove spatial gradients in the blind pointing errors, so these must be small in both azimuth and elevation. In practice, the elevation error is larger, so its gradient will determine the tracking time (Condon 2003c). If we require that the blind elevation error  $\Delta E$  not contribute more than 1/4 of the total pointing error  $\sigma_2^2$  when averaged over the time  $\Delta t$  between successive calibrations, then

$$\frac{1}{\Delta t} \int_0^{\Delta t} (\Delta E)^2 = \frac{\sigma_2^2}{4}. \quad (18)$$

The maximum permissible elevation gradients in the blind pointing consistent with Equations 11 and 18 are listed in Table 1. The elevation gradient has been reduced significantly by the new “traditional” pointing model and thermal corrections; it is now  $< 10'' \text{ rad}^{-1}$  for elevations  $E > 15^\circ$ . This is good enough for tracking times up to  $\Delta t = 0.5$  hours at 52 GHz. The final PTCS requirements will be to satisfy Equation 18 for good pointing at 86 GHz ( $6''.5 \text{ rad}^{-1}$ ) and usable pointing at 115 GHz ( $7''.0 \text{ rad}^{-1}$ ) above  $E = 15^\circ$ .

## 4. Wavefront Phase Irregularities

Viewed as a transmitting antenna, the GBT should emit a plane wave normal to the line-of-sight. De-focusing as well as deviations of the primary and secondary reflector surfaces from their ideal shapes introduce wavefront phase errors that reduce gain and aperture efficiency, broaden the main beam, and increase sidelobe levels. The contributions from these errors can be combined and treated as a single “effective surface error” in the aperture plane.

### 4.1. Efficiency and the Effective Surface Error

If the rms deviation from the ideal aperture is  $\epsilon$  and the error correlation length is much smaller than the aperture diameter, the aperture efficiency is multiplied by the factor (Ruze 1966)

$$\eta_s \approx \exp \left[ - \left( \frac{4\pi\epsilon}{\lambda} \right)^2 \right]. \quad (19)$$

The traditional requirement for the efficiency  $\eta_s$  of a “good” aperture is

$$\epsilon < \frac{\lambda}{16} \quad \text{so} \quad \eta_s \approx 0.54 . \quad (20)$$

A looser criterion for “usable” performance is that the forward gain, which is proportional to  $\eta_s/\lambda^2$ , not decline as wavelength decreases. It implies

$$\epsilon < \frac{\lambda}{4\pi} \quad \text{so} \quad \eta_s \approx 0.37 . \quad (21)$$

The GBT has two mirrors, both of which are imperfect, and collimation errors may not be negligible. Even so, we can use Equation 19 to *define* an effective rss (root sum square) surface error  $\epsilon$  that incorporates all of these contributions. At  $\nu = 52$  GHz we get  $\epsilon = 0.36$  mm from the  $\epsilon < \lambda/16$  criterion (Eq. 20), in agreement with the requirement specified by Lockman (1998). Hall et al. (1993) specify rss surface accuracies of 0.42 mm and 0.22 mm for Phases II and III; these correspond to frequency limits of 45 GHz and 85 GHz for good efficiency. Table 1 lists the rms errors  $\epsilon$  needed for both “usable” and “good” performance at 52, 86, and 115 GHz. The GBT currently appears to be usable at 52 GHz (Condon 2003c). The PTCS requirement for early 2005 is a usable surface at 86 GHz ( $\epsilon = 0.28$  mm). The requirement for October 2005 is a “good” surface at 86 GHz and a “usable” surface at 115 GHz, both implying  $\epsilon \approx 0.21$  mm.

## 4.2. Axial Focusing

The asymmetrical GBT actually has two separate axial foci: (1) The subreflector location  $y_\pi$  yields minimum cross-polarization. This position was chosen to be the nominal origin of the  $y_s$  axis (Srikanth 1990b); that is  $y_\pi \equiv 0$ . (2) The subreflector location  $y_\gamma$  maximizes the forward gain. It is located slightly below the best polarization focus, at  $y_\gamma \approx -0.43\lambda$ . Electromagnetic simulations of the GBT (Srikanth 1990b) indicate that we may approximate the variation of relative power gain  $g_a$  resulting from axial defocusing by

$$g_a = \exp \left[ -4 \ln(2) \left( \frac{\Delta y_s}{\theta_A} \right)^2 \right] , \quad (22)$$

where  $\Delta y_s \equiv (y_s - y_\gamma)$  is the axial displacement of the subreflector from the optimum-gain focus. Equation 22 is a Gaussian with a FWHM “beamwidth”  $\theta_A \approx 4\lambda$  and a maximum at  $y_s = y_\gamma$ . Choosing the best polarization focus ( $\Delta y_s \approx +0.43\lambda$ ) yields  $g_f \approx 0.97$ ; that is, there is a 3% penalty in power gain.

How accurately do we need to set the axial focus? If we demand that  $g_a > 0.99$  for “good” focusing, Equation 22 indicates that the axial focusing error should be less than  $\theta_A/16$ , or

$$|\Delta y_s| < \lambda/4 . \quad (23)$$

This gain criterion is actually stricter than the  $|\Delta y_s| < \lambda/10$  requirement used at the  $f/D = 0.35$ ,  $D = 30$  m IRAM telescope since the axial focusing “beamwidth” in wavelengths is roughly proportional to  $(f/D)^2$  and independent of  $D$ . If the strict focusing requirement (Equation 23) proves too onerous at short wavelengths (because too much observing time would be lost refocusing the GBT frequently, for

example), it could be relaxed significantly. Comparing Equations 2 and 22 makes it clear that the effects of pointing and defocusing on gain are similar, so that we can apportion the total allowed gain loss more equally between them. A reasonable choice guided by our pointing criterion might be to require only that  $g_a > 0.95$ , which can be satisfied by the relaxed requirement

$$|\Delta y_s| < \lambda/2 \quad (24)$$

for “usable” axial focusing.

The current offset focusing results from the GBT are quite promising, easily surpassing the requirement for usable operation at 52 GHz (Table 1). We should easily reach usable performance at 86 GHz ( $|\Delta y_s| = 1.7$  mm) in early 2005. The final PTCS requirement for October 2005 is usable performance at 115 GHz ( $|\Delta y_s| = 1.3$  mm), and we have set ourselves a goal of good performance at 86 GHz ( $|\Delta y_s| = 0.9$  mm).

Table 1. PTCS Specifications and Constraints

Parameter	$\nu = 52$ GHz	$\nu = 86$ GHz	$\nu = 115$ GHz	Units	Notes
Wavelength $\lambda$	5.8	3.5	2.6	mm	
FWHM beam $\theta$	14.2	8.6	6.4	arcsec	Eq. 1
“Usable” offset tracking $\sigma_2$	2.8	1.7	1.3	arcsec	Eq. 6
current wind $ v $	3.0 (6.6)	2.3 (5.2)	2.0 (4.5)	$\text{m s}^{-1}$ (mph)	Eqs. 6, 12, 13
requirement wind $ v $	3.5 (7.8)	2.5 (5.6)	2.5 (5.6)	$\text{m s}^{-1}$ (mph)	
“Good” offset tracking $\sigma_2$	2.0	1.2	...	arcsec	Eqs. 7 and 12
current wind $ v $	2.5 (5.6)	1.9 (4.3)	...	$\text{m s}^{-1}$ (mph)	Eqs. 7, 12, 13
requirement wind $ v $	2.5 (5.6)	2.5 (5.6)	...	$\text{m s}^{-1}$ (mph)	
Calibrator accuracy $\sigma_2(\text{cal})$	0.7	0.3	0.3	arcsec	Eq. 8
Tracking time $\Delta t$	0.5	0.5	0.5	hours	Eq. 11
Blind azimuth pointing $\sigma_A$	4.3	2.6	1.9	arcsec	Eq. 16
Blind pointing gradient $ \Delta E /\text{rad}$	10.8	6.5	7.0	"/rad	Eq. 18
“Usable” effective $\epsilon$	0.46	0.28	0.21	mm	Eq. 21
“Usable” focusing $ \Delta y_s $	2.9	1.7	1.3	mm	Eq. 23
“Good” effective $\epsilon$	0.36	0.22	...	mm	Eq. 20
“Good” focusing $ \Delta y_s $	1.4	0.9	...	mm	Eq. 24

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